Lecture 11

Introduction to Machine Learning

EECS 398: Practical Data Science, Winter 2025

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Agenda 📆

- Machine learning and models.
- The constant model.
- Minimizing mean squared error using calculus.
- Another loss function.

The next few lectures (and Homework 6!) will be primarily **math-based**.

- For these lectures, we'll post blank slides as a PDF before class, and annotated slides after class.
- If there are any code demos, we'll post those before class, too.

Machine learning and models

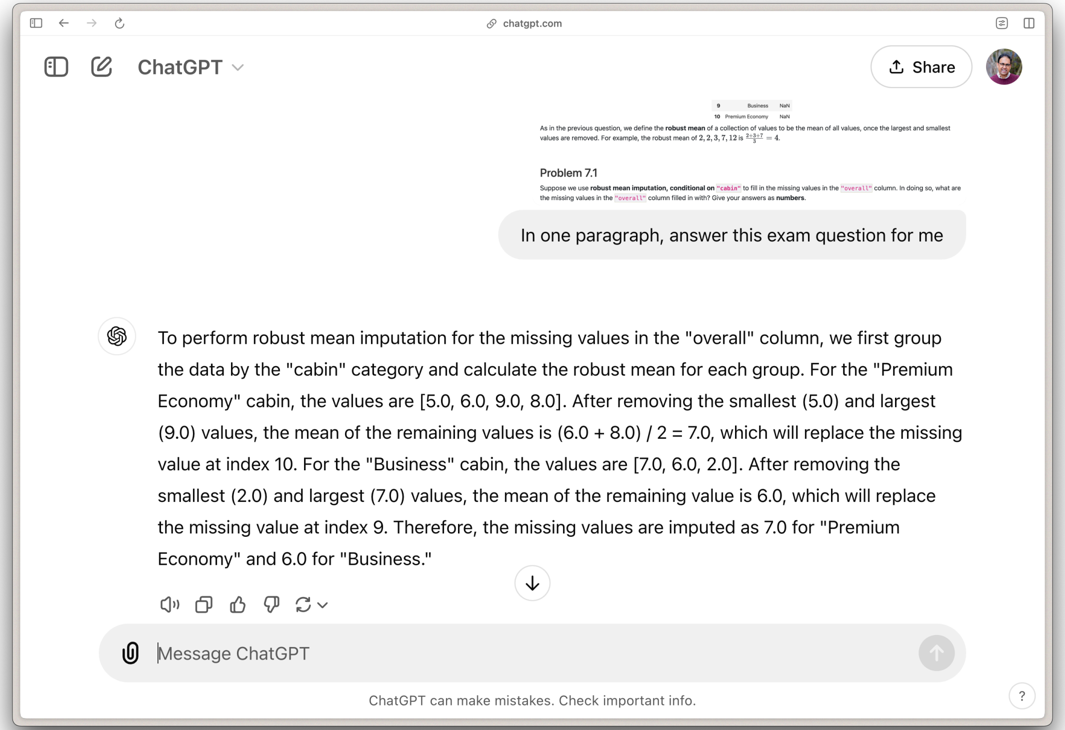
Machine learning is about **automatically** learning patterns from data.

Example: Handwritten digit classification

Humans are good at understanding handwriting,

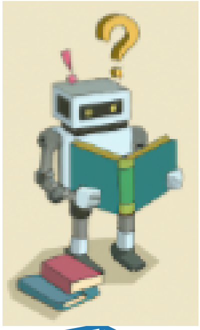
but how do we get computers to understand handwriting?

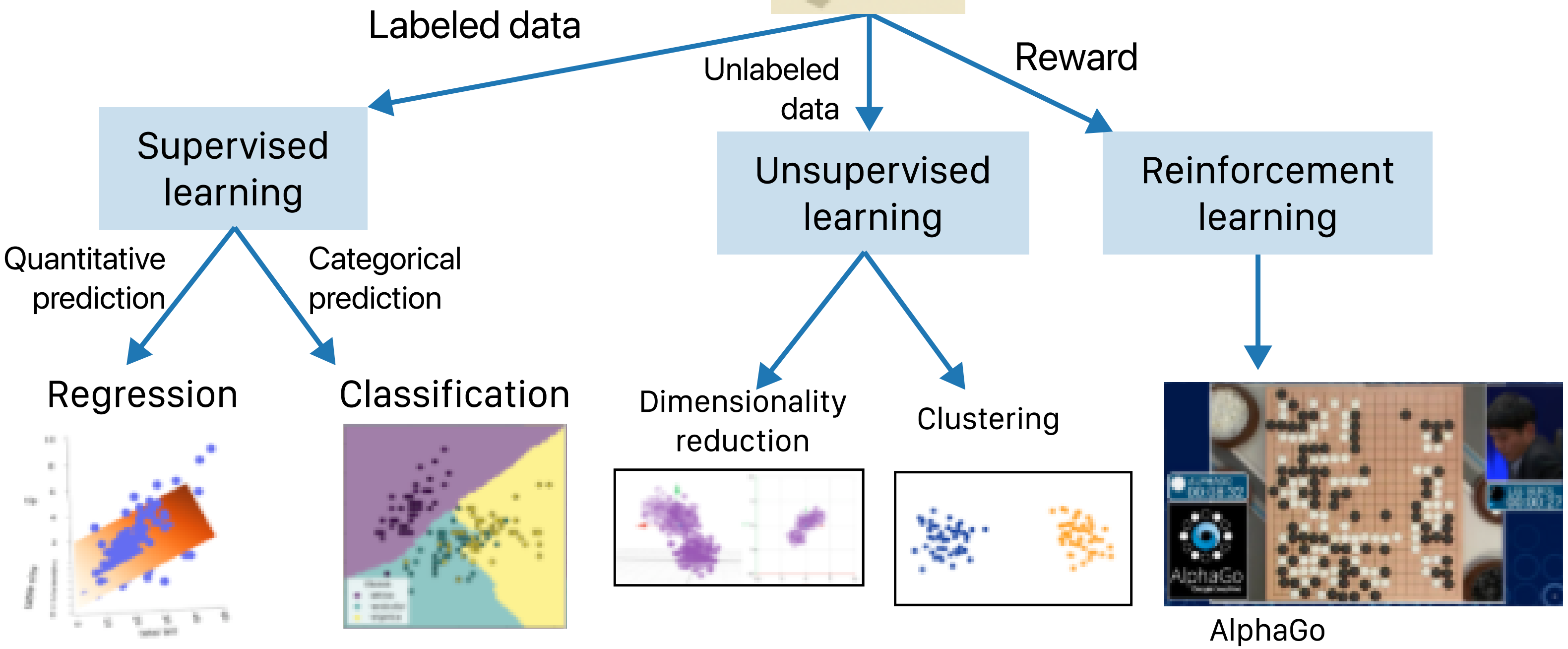
Example: ChatGPT

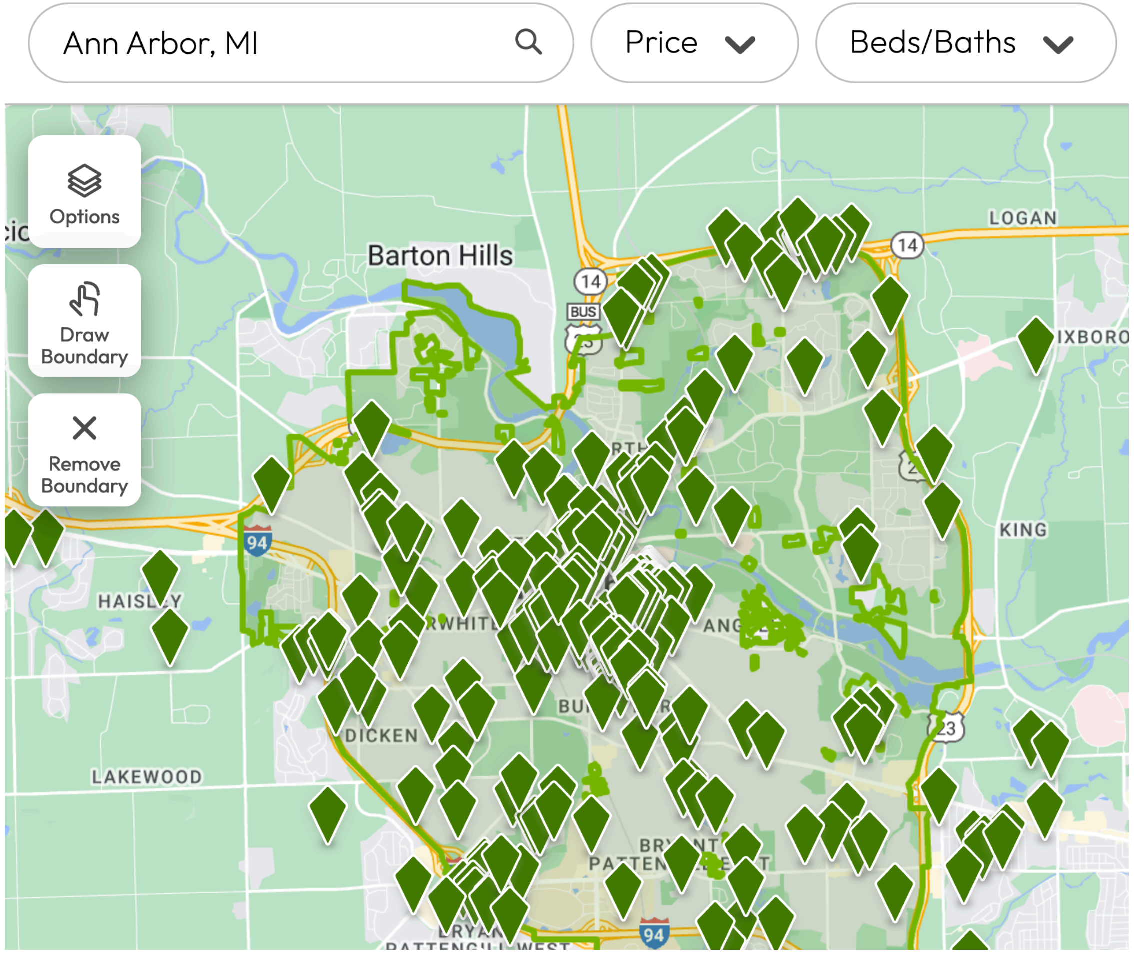


How did ChatGPT know how to answer Question 7 from the Fall 2024 Midterm?

Taxonomy of machine learning







You might be starting to look for off-campus apartments for next year, none of which are in your price range.

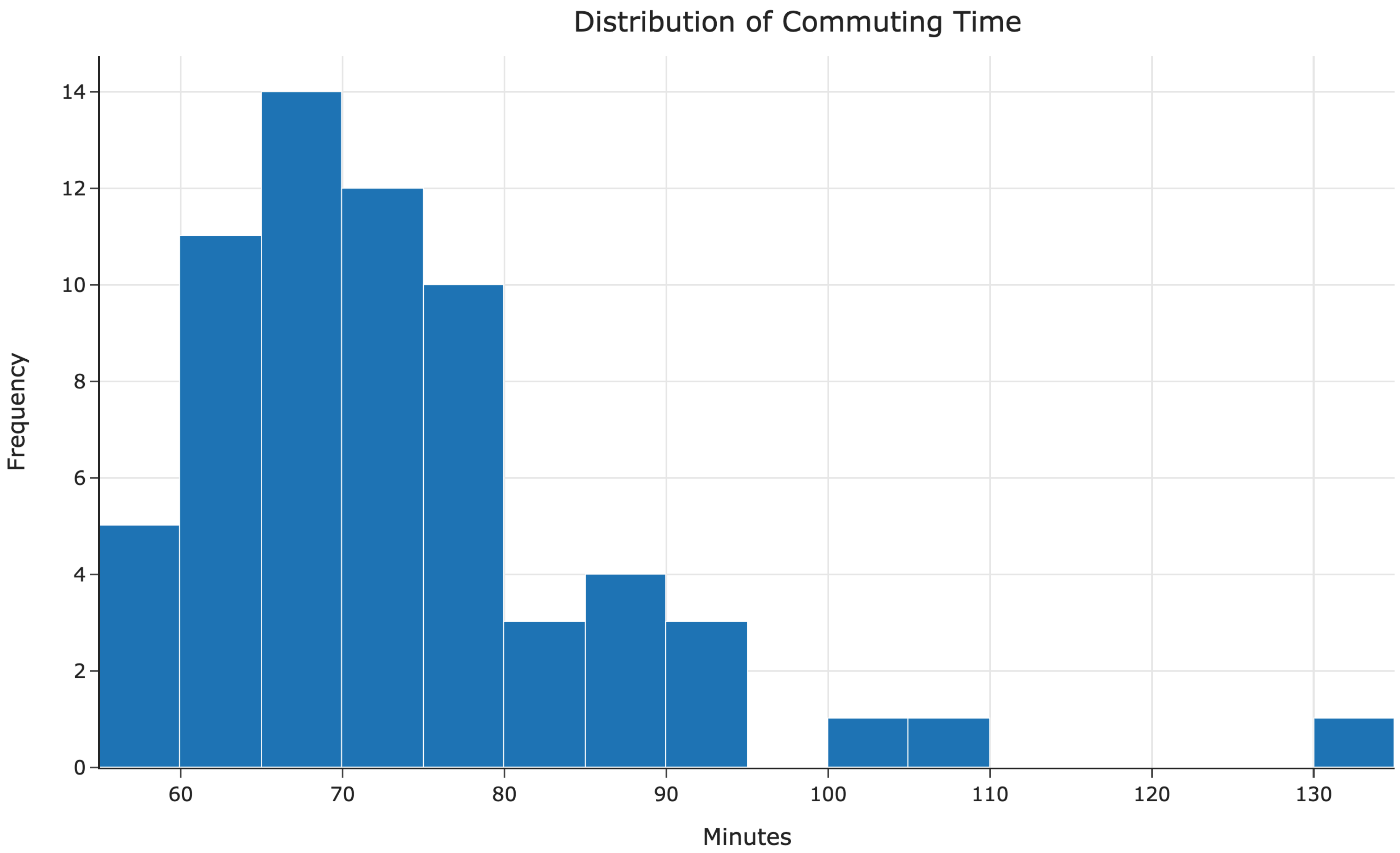
	date	day	departure_hour	minutes
0	5/22/2023	Mon	8.450000	63.0
1	9/18/2023	Mon	7.950000	75.0
2	10/17/2023	Tue	10.466667	59.0
3	11/28/2023	Tue	8.900000	89.0
4	2/15/2024	Thu	8.083333	69.0

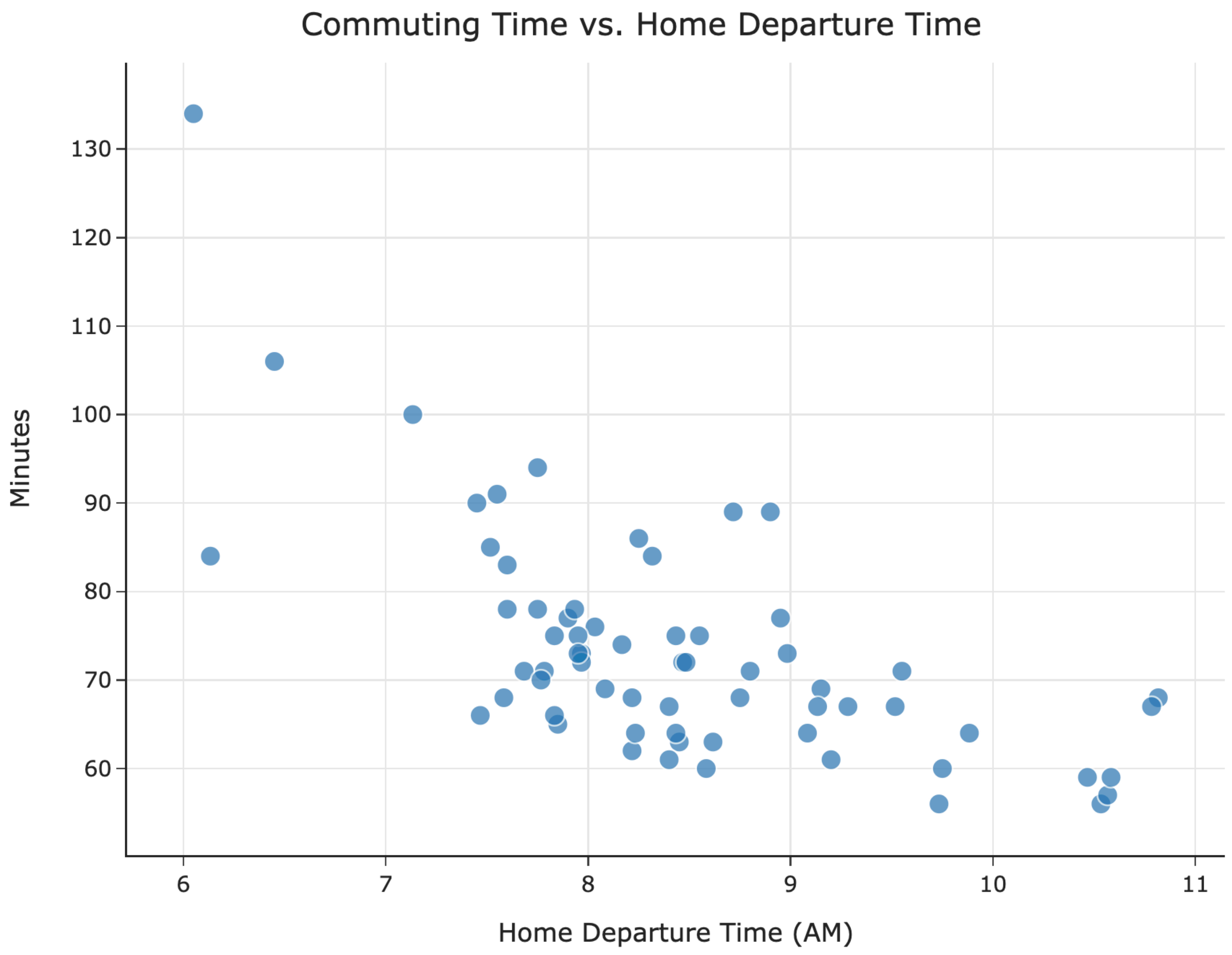
...

You decide to live with your parents in Detroit and commute.

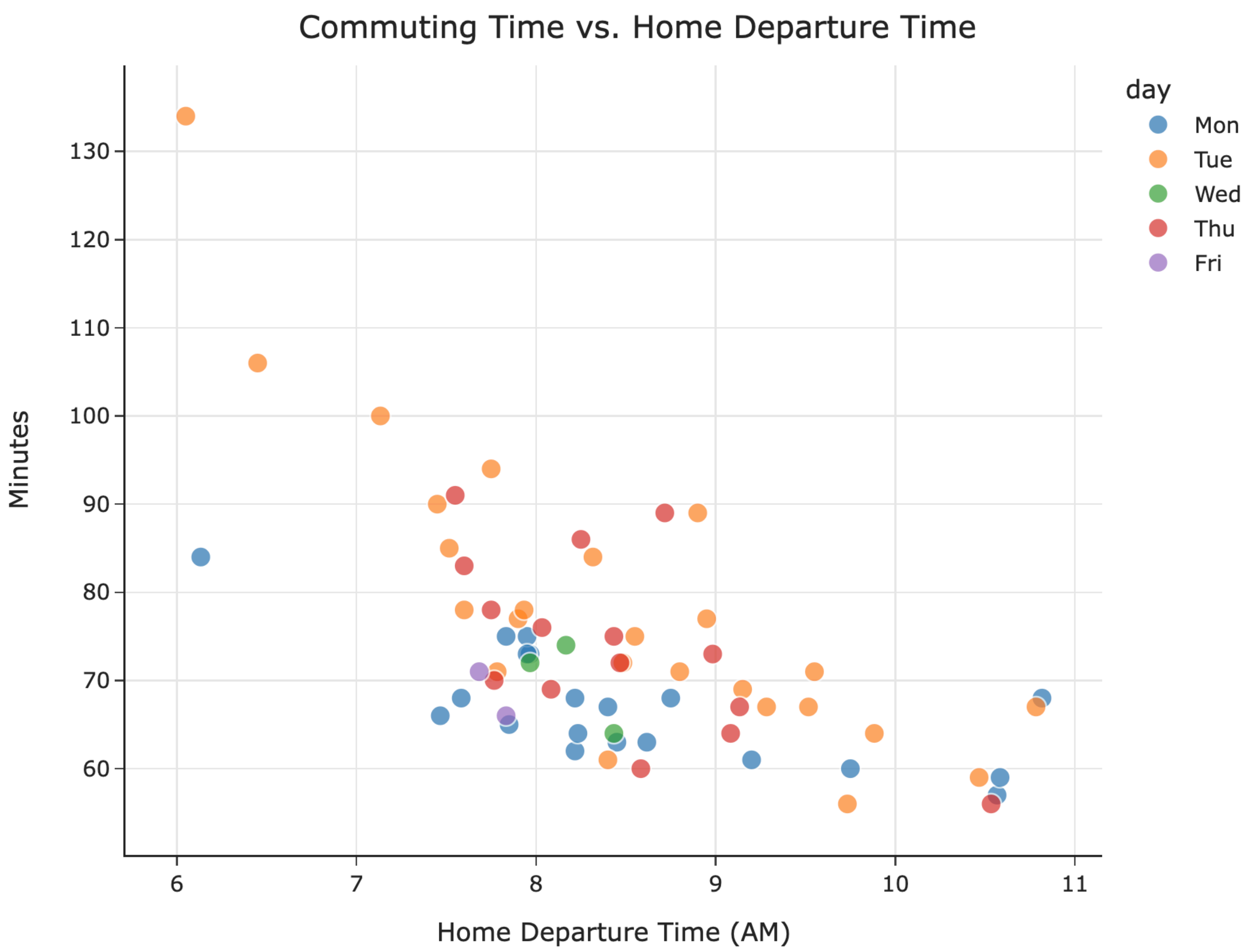
You keep track of how long it takes you to get to school each day.

This is a real dataset, collected by Joseph Hearn! However, he lived in the Seattle area, not Metro Detroit.


Distribution of Commuting Time
Frequency
14
12
10
8
6
4
2
0
60
70
80
90
100
110
120
130
Minutes



Commuting Time vs. Home Departure Time
Minutes
130
120
110
100
90
80
70
60
6
7
8
9
10
11
Home Departure Time (AM)



Commuting Time vs. Home Departure Time
day
Mon
Tue
Wed
Thu
Fri
Minutes
130
120
110
100
90
80
70
60
6
7
8
9
10
11
Home Departure Time (AM)

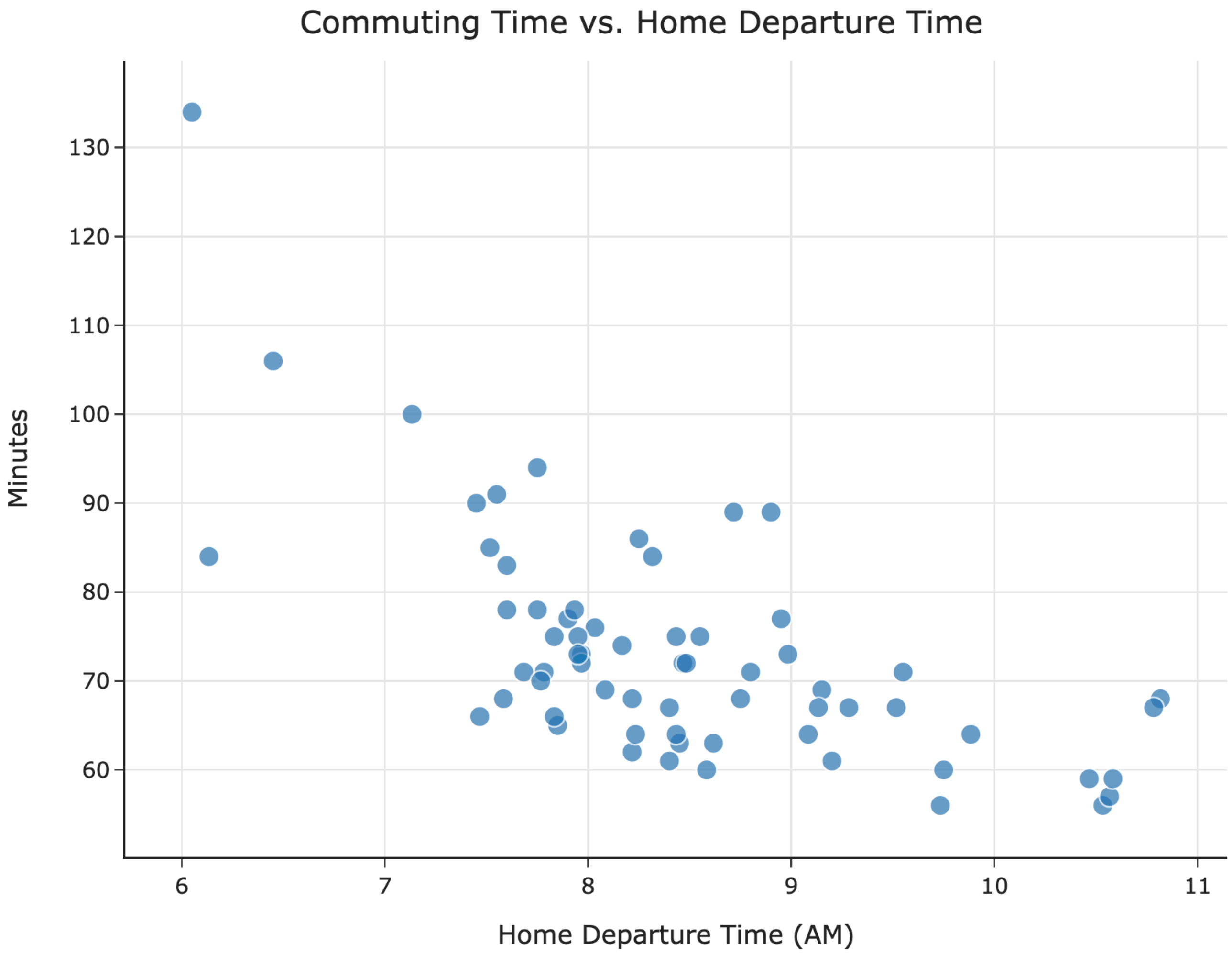
Goal: Predict your **commute time**, i.e. how long it will take to get to school.

This is a **regression** problem.

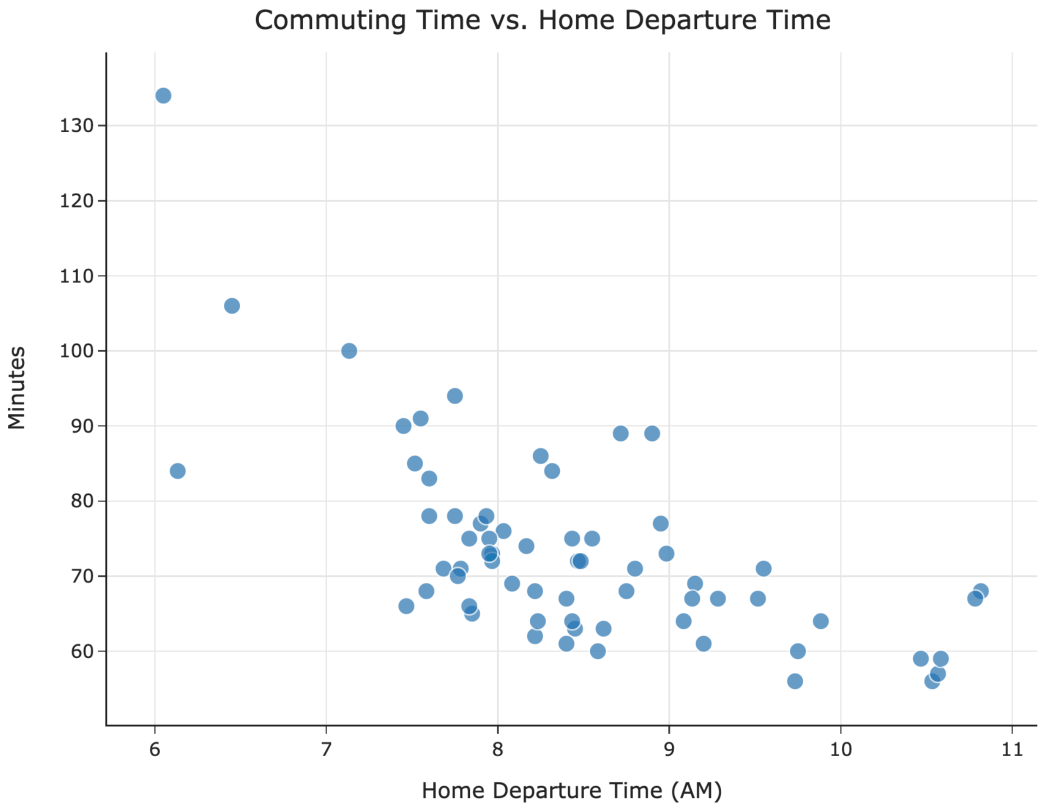
How can we do this? What will we need to assume?

A **model** is a set of assumptions about how data were generated.

Possible models



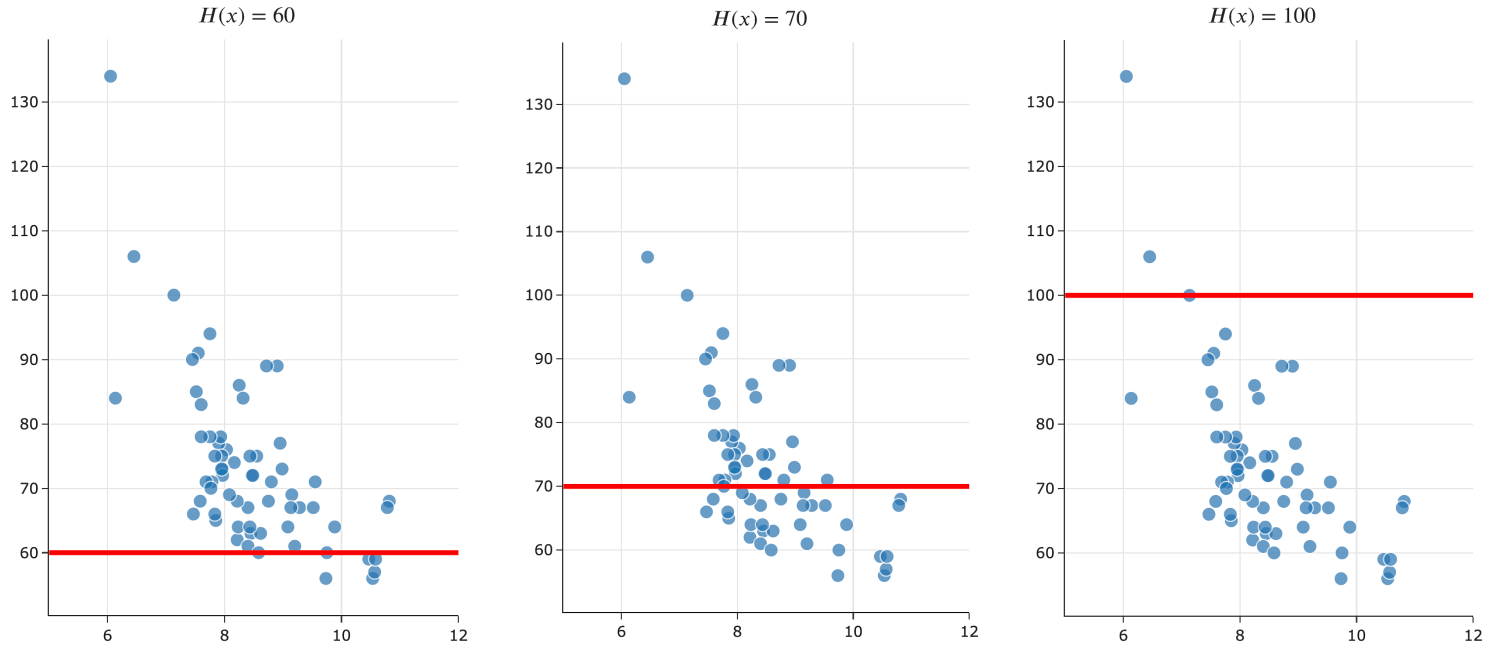
Notation



- x: "input", "independent variable", or "feature".
- y: "response", "dependent variable", or "target".
- The ith observation is denoted (x_i, y_i).
- **We use x_is to predict y_is.**

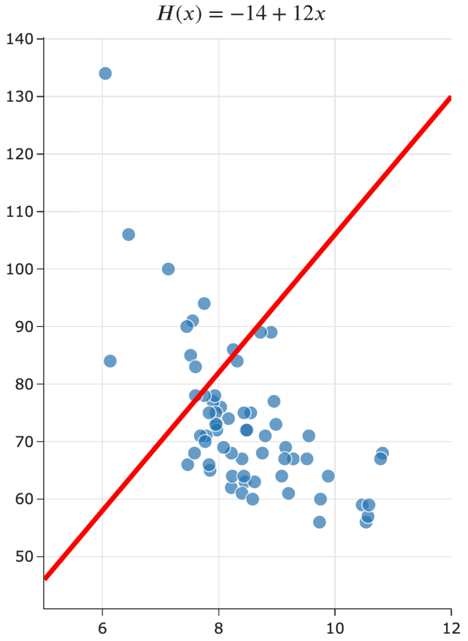
Hypothesis functions and parameters

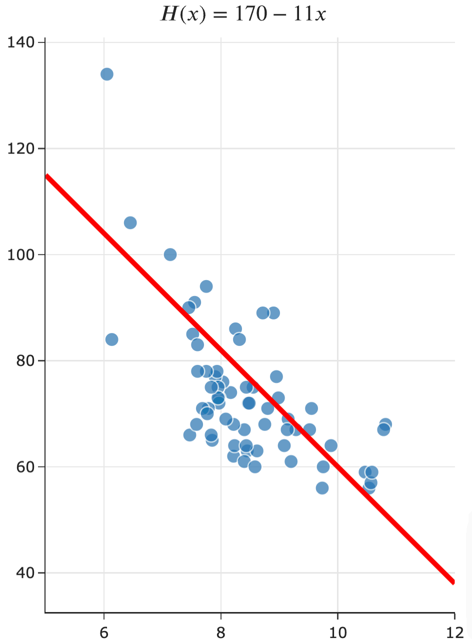
- A hypothesis function, H, takes in an x_i as input and returns a predicted y_i.
- **Parameters** define the relationship between the input and output of a hypothesis function.
- **Example**: The constant model, $H(x_i) = h$, has one parameter: h.



Hypothesis functions and parameters

- A hypothesis function, H, takes in an x_i as input and returns a predicted y_i.
- **Parameters** define the relationship between the input and output of a hypothesis function.
- **Example**: The simple linear regression model, $H(x_i) = w_0 + w_1 x_i$, has two parameters: w_0 and w_1.





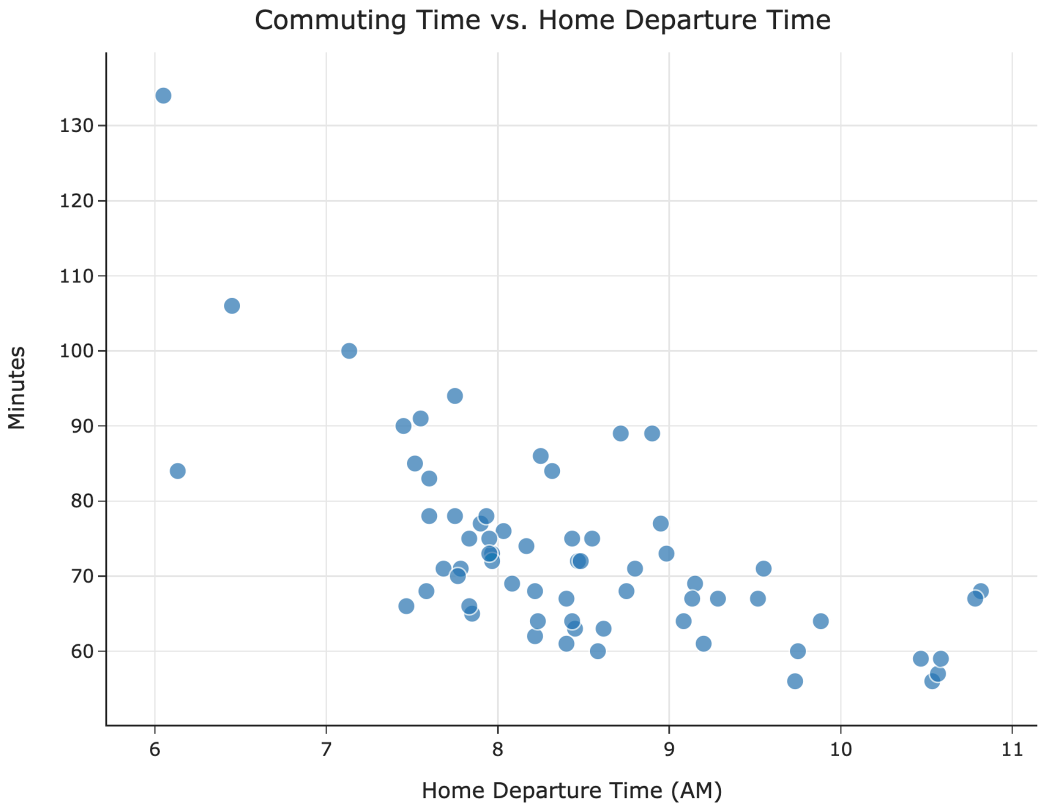
Question 🤔

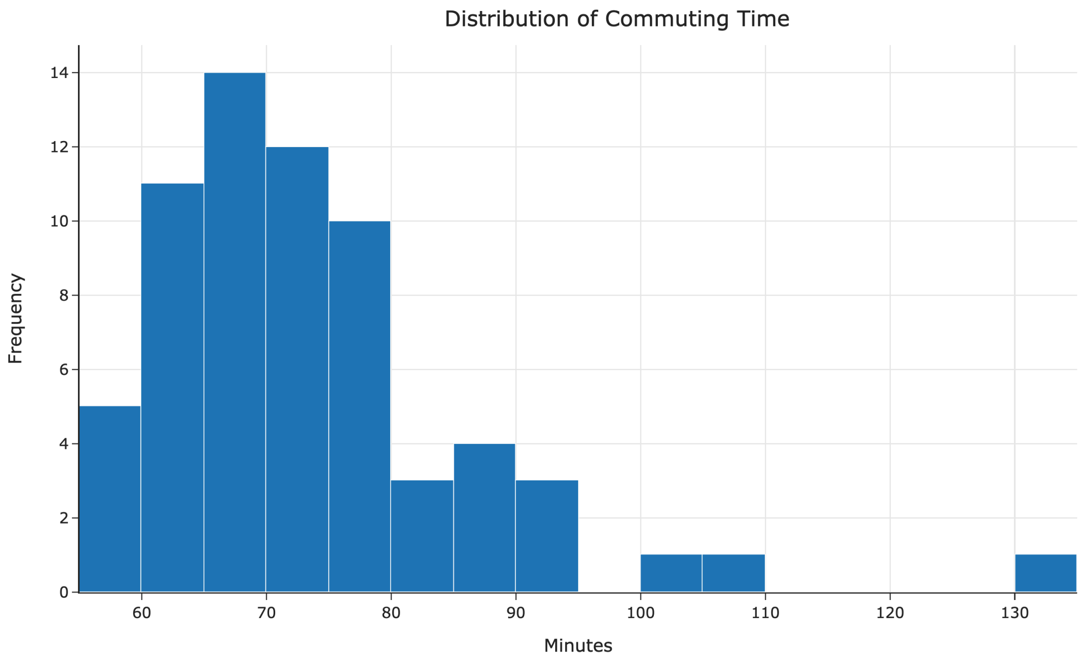
Answer at practicaldsc.org/q

What questions do you have?

The constant model

The constant model





A concrete example

- Let's suppose we have just a smaller dataset of just five historical commute times in minutes.

$$
\begin{aligned}
y_1 &= 72 \\
y_2 &= 90 \\
y_3 &= 61 \\
y_4 &= 85 \\
y_5 &= 92
\end{aligned}
$$

- Given this data, can you come up with a prediction for your future commute time? How?

Some common approaches

- The **mean**:

$$\frac{1}{5}(72 + 90 + 61 + 85 + 92) = \boxed{80}$$

- The **median**:

$$61 \qquad 72 \qquad \boxed{85} \qquad 90 \qquad 92$$

- Both of these are familiar **summary statistics**.

  Summary statistics summarize a collection of numbers with a single number, i.e. they result from an **aggregation**.

- But which one is better? Is there a "best" prediction we can make?

The cost of making predictions

- A **loss function** quantifies how bad a prediction is for a single data point.
  - If our prediction is **close** to the actual value, we should have **low** loss.
  - If our prediction is **far** from the actual value, we should have **high** loss.
- A good starting point is error, which is the difference between **actual** and **predicted** values.

$$e_i = y_i - H(x_i)$$

- Suppose my commute **actually** takes 80 minutes.
  - If I predict 75 minutes:
  - If I predict 72 minutes:
  - If I predict 100 minutes:

Squared loss

- One loss function is squared loss, L_{sq}, which computes $(\text{actual} - \text{predicted})^2$.

$$L_{\text{sq}}(y_i, H(x_i)) = (y_i - H(x_i))^2$$

- Note that for the constant model, $H(x_i) = h$, so we can simplify this to:

$$L_{\text{sq}}(y_i, h) = (y_i - h)^2$$

- Squared loss is not the only loss function that exists!

  Soon, we'll learn about absolute loss. Different loss functions have different pros and cons.

A concrete example, revisited

- Consider again our smaller dataset of just five historical commute times in minutes.

$$y_1 = 72$$
$$y_2 = 90$$
$$y_3 = 61$$
$$y_4 = 85$$
$$y_5 = 92$$

- Suppose we predict the median, $h = 85$. What is the squared loss of 85 for each data point?

Averaging squared losses

- We'd like a single number that describes the quality of our predictions across our entire dataset. One way to compute this is as the **average of the squared losses**.
- For the median, $h = 85$:

$$\frac{1}{5}\left((72-85)^2 + (90-85)^2 + (61-85)^2 + (85-85)^2 + (92-85)^2\right) = \boxed{163.8}$$

- For the mean, $h = 80$:

$$\frac{1}{5}\left((72-80)^2 + (90-80)^2 + (61-80)^2 + (85-80)^2 + (92-80)^2\right) = \boxed{138.8}$$

- Which prediction is better? Could there be an even better prediction?

Mean squared error

- Another term for **<u>average</u> squared loss** is **<u>mean</u> squared error (MSE)**.
- The mean squared error on our smaller dataset for any prediction h is of the form:

$$R_{\text{sq}}(h) = \frac{1}{5}\left((72-h)^2 + (90-h)^2 + (61-h)^2 + (85-h)^2 + (92-h)^2\right)$$

R stands for "risk", as in "empirical risk." We'll see this term again soon.

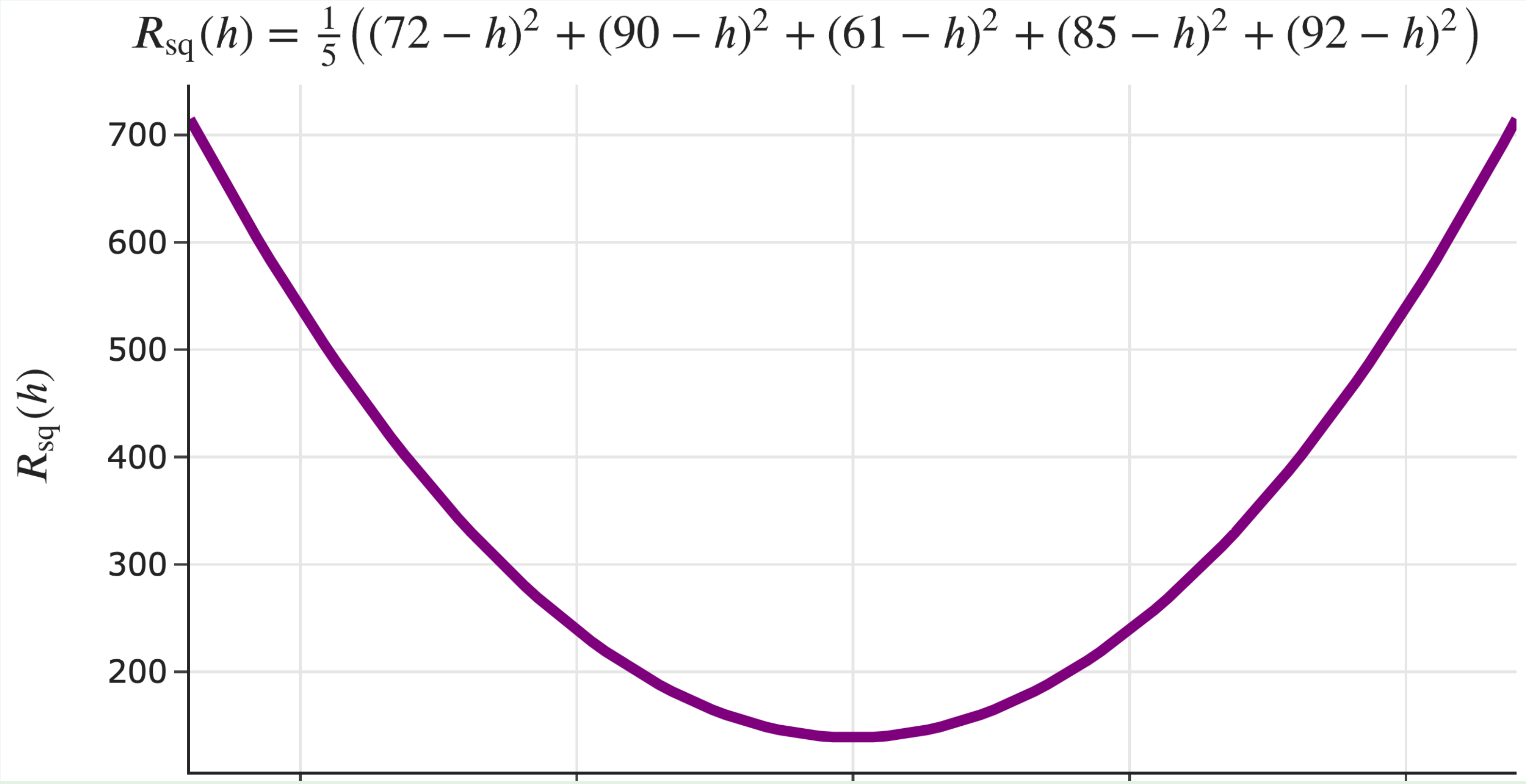
- For example, if we predict $h = 100$, then:

$$\begin{aligned} R_{\text{sq}}(100) &= \frac{1}{5}\left((72-100)^2 + (90-100)^2 + (61-100)^2 + (85-100)^2 + (92-100)^2\right) \\ &= \boxed{538.8} \end{aligned}$$

- **We can pick any h as a prediction, but the smaller $R_{\text{sq}}(h)$ is, the better h is!**

Activity

Answer at practicaldsc.org/q (use the free response box!)



Which h corresponds to the vertex of $R_{sq}(h)$?

Mean squared error, in general

- Suppose we collect n commute times, $y_1, y_2, \ldots, y_n$.
- The mean squared error of the prediction h is:

- Or, using **summation notation**:

The best prediction

$$R_{\text{sq}}(h) = \frac{1}{n}\sum_{i=1}^{n}(y_i - h)^2$$

- We want the **best** constant prediction, among all constant predictions h.
- The smaller $R_{\text{sq}}(h)$ is, the better h is.
- **Goal**: Find the h that minimizes $R_{\text{sq}}(h)$.

  The resulting h will be called h^*.
- **How do we find h^*?**

Minimizing mean squared error using calculus

Minimizing using calculus

- We'd like to minimize:

$$R_{\text{sq}}(h) = \frac{1}{n}\sum_{i=1}^{n}(y_i - h)^2$$

- In order to minimize $R_{\text{sq}}(h)$, we:
  1. take its derivative with respect to h,
  2. set it equal to 0,
  3. solve for the resulting h^*, and
  4. perform a second derivative test to ensure we found a minimum.
- $R_{\text{sq}}(h)$ is an example of an **objective function**, a function that needs to be minimized.

Step 0: The derivative of $(y_i - h)^2$

- Remember from calculus that:
  - if $c(x) = a(x) + b(x)$, then
  - $\frac{d}{dx}c(x) = \frac{d}{dx}a(x) + \frac{d}{dx}b(x)$.
- This is relevant because $R_{\text{sq}}(h) = \frac{1}{n}\sum_{i=1}^{n}(y_i - h)^2$ involves the sum of n individual terms, each of which involve h.
- So, to take the derivative of $R_{\text{sq}}(h)$, we'll first need to find the derivative of $(y_i - h)^2$.

$$\frac{d}{dh}(y_i - h)^2 =$$

Question 🤔

Answer at practicaldsc.org/q

$$R_{\text{sq}}(h) = \frac{1}{n} \sum_{i=1}^{n} (y_i - h)^2$$

Which of the following is $\frac{d}{dh} R_{\text{sq}}(h)$?

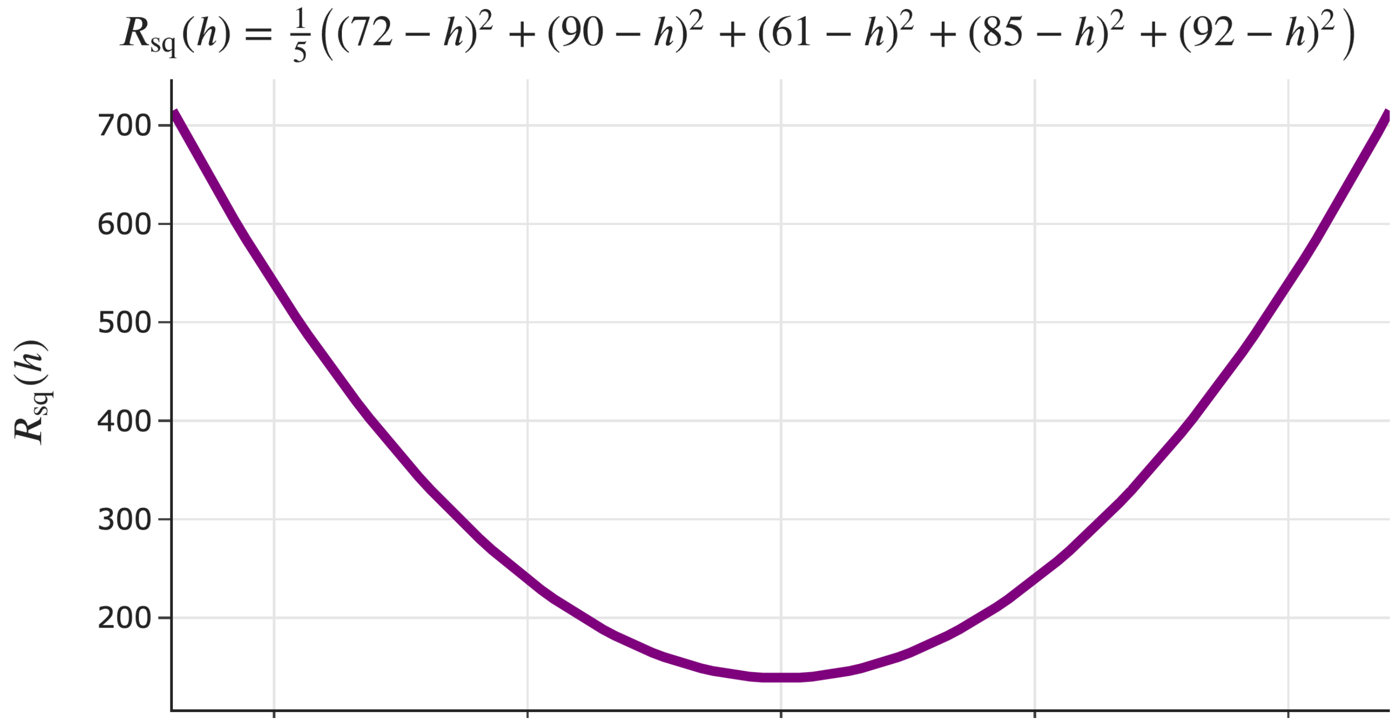
- A. 0
- B. $\sum_{i=1}^{n} y_i$
- C. $\frac{1}{n} \sum_{i=1}^{n} (y_i - h)$
- D. $\frac{2}{n} \sum_{i=1}^{n} (y_i - h)$
- E. $-\frac{2}{n} \sum_{i=1}^{n} (y_i - h)$

Step 1: The derivative of $R_{\text{sq}}(h)$

$$\frac{d}{dh} R_{\text{sq}}(h) = \frac{d}{dh} \left(\frac{1}{n} \sum_{i=1}^{n} (y_i - h)^2 \right)$$

Steps 2 and 3: Set to 0 and solve for the minimizer, h^*

Step 4: Second derivative test



We already saw that $R_{\text{sq}}(h)$ is **convex**, i.e. that it opens upwards, so the h^* we found must be a minimum, not a maximum.

The mean minimizes mean squared error!

- The problem we set out to solve was, find the h^* that minimizes:

$$R_{\text{sq}}(h) = \frac{1}{n} \sum_{i=1}^{n} (y_i - h)^2$$

- The answer is:

$$h^* = \text{Mean}(y_1, y_2, \ldots, y_n)$$

- The **best constant prediction**, in terms of mean squared error, is always the **mean**.
- We call h^* our **optimal model parameter**, for when we use:
  - the constant model, $H(x_i) = h$, and
  - the squared loss function, $L_{\text{sq}}(y_i, h) = (y_i - h)^2$.

Aside: Terminology

- Another way of writing:

$$h^* \text{ is the value of } h \text{ that minimizes } \frac{1}{n}\sum_{i=1}^{n}(y_i - h)^2$$

  is:

$$h^* = \underset{h}{\text{argmin}} \left(\frac{1}{n}\sum_{i=1}^{n}(y_i - h)^2 \right)$$

- h^* is the solution to an **optimization problem**, where the objective function is $R_{\text{sq}}(h) = \frac{1}{n}\sum_{i=1}^{n}(y_i - h)^2$.

The modeling recipe

- We've implicitly introduced a three-step process for finding optimal model parameters (like h^*) that we can use for making predictions:

    1. Choose a model.

    2. Choose a loss function.

    3. Minimize average loss to find optimal model parameters.

- Most modern machine learning methods today, including neural networks, follow this recipe, and we'll see it repeatedly this semester!