Matrices

Matrices

- An $n \times d$ matrix is a table of numbers with n rows and d columns.
- We use upper-case letters to denote matrices.

$$A=\begin{bmatrix}2&5&8\\-1&5&-3\end{bmatrix}_{\textbf{2\times3}} \qquad \text{the set of matrices with 2 rows and 3 columns}$$
 • Since A has two rows and three columns, we say $A\in\mathbb{R}^{2\times3}$.

• Key idea: Think of a matrix as several column vectors, stacked next to each other.

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \end{bmatrix} \begin{bmatrix} 8 \\ -3 \end{bmatrix}$$

Matrix addition and scalar multiplication

- We can add two matrices only if they have the same dimensions.
- Addition occurs elementwise:

$$egin{bmatrix} 2 & 5 & 8 \ -1 & 5 & -3 \end{bmatrix} + egin{bmatrix} 1 & 2 & 3 \ 0 & 1 & 2 \end{bmatrix} = egin{bmatrix} 3 & 7 & 11 \ -1 & 6 & -1 \end{bmatrix}$$

• Scalar multiplication occurs elementwise, too:

$$2\begin{bmatrix} 2 & 5 & 8 \\ -1 & 5 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 10 & 16 \\ -2 & 10 & -6 \end{bmatrix}$$

Matrix-matrix multiplication

• Key idea: We can multiply matrices A and B if and only if:

$$\# ext{ columns in } A = \# ext{ rows in } B$$

- If A is n imes d and B is d imes p, then AB is n imes p.
- Example: If A is as defined below, what is A^TA ?

$$A = \begin{bmatrix} 2 & 5 & 8 \\ -1 & 5 & -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ 5 & 5 \end{bmatrix}$$

$$A^{T}A = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$$

$$A^{T}A = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$$

$$A^{T}A = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$$

$$A^{T}A = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$$

Question 🤔

Answer at q.dsc40a.com

Assume A, B, and C are all matrices. Select the **incorrect** statement below.

• A.
$$A(B+C) = AB + AC$$
.

• B.
$$A(BC) = (AB)C$$
.

$$\bullet$$
 C. $AB = BA$.

• D.
$$(A + B)^T = A^T + B^T$$
.

• E.
$$(AB)^T = B^T A^T$$
.

Matrix-vector multiplication

• A vector $ec{v} \in \mathbb{R}^n$ is a matrix with n rows and 1 column.

$$ec{v} = egin{bmatrix} v_1 \ v_2 \ dots \ v_n \end{bmatrix}$$

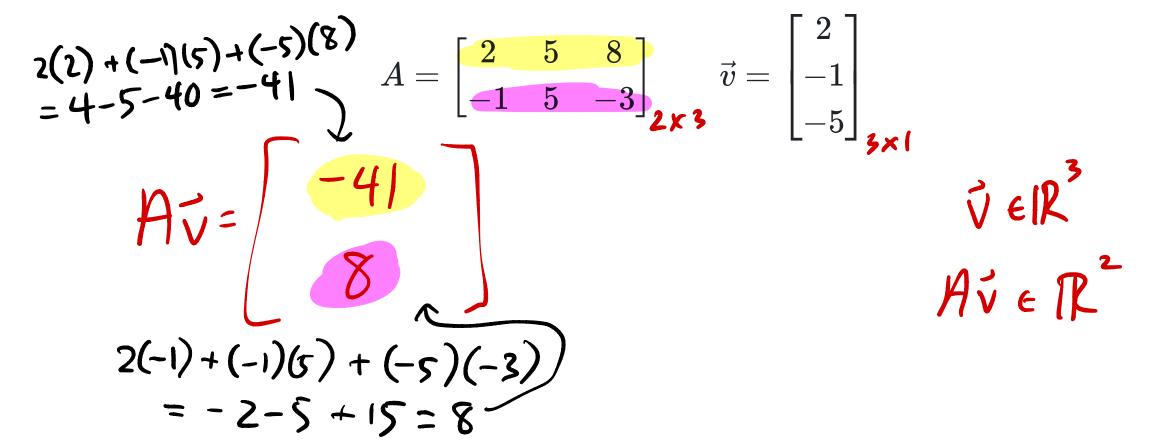
- Suppose $A \in \mathbb{R}^{n \times d}$.

$$\text{ What must the dimensions of } \vec{v} \text{ be in order for the product } A\vec{v} \text{ to be valid?}$$

 \circ What must the dimensions of $ec{v}$ be in order for the product $ec{v}^T A$ to be valid?

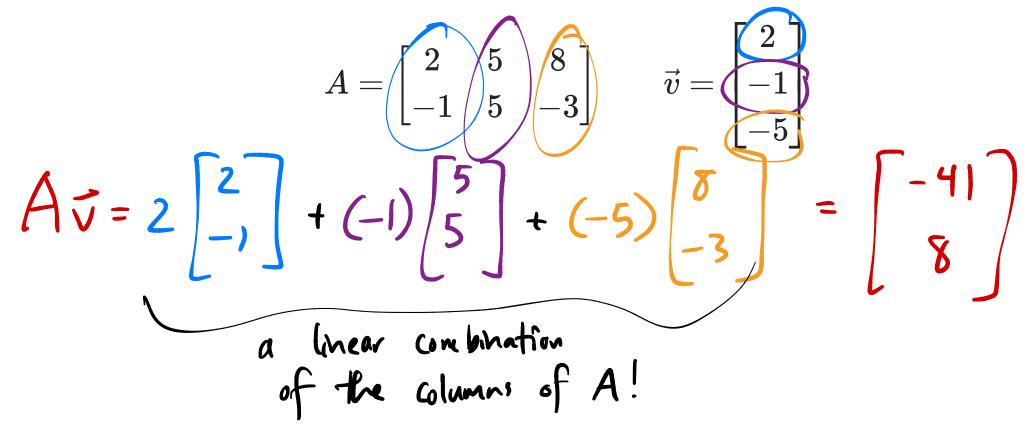
One view of matrix-vector multiplication

- One way of thinking about the product $A\vec{v}$ is that it is **the dot product of** \vec{v} **with every** row of A.
- Example: What is $A\vec{v}$?



Another view of matrix-vector multiplication

- Another way of thinking about the product $A\vec{v}$ is that it is a linear combination of the columns of A, using the weights in \vec{v} .
- Example: What is $A\vec{v}$?



Matrix-vector products create linear combinations of columns!

• **Key idea**: It'll be very useful to think of the matrix-vector product $A\vec{v}$ as a linear combination of the columns of A, using the weights in \vec{v} .

