d vectors,
each has n components

Linear combinations

- Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d$ all be vectors in $\mathbb{R}^n$.
- A **linear combination** of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d$ is any vector of the form:

$$a_1\vec{v}_1 + a_2\vec{v}_2 + \ldots + a_d\vec{v}_d$$

where $a_1, a_2, \ldots, a_d$ are all scalars.

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} -1 \\ 4 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 9 \end{bmatrix}$$

Examples

$$2\vec{v} + \vec{v}_2 + \frac{1}{9}\vec{v}_3 = \begin{bmatrix} \_ \\ \_ \end{bmatrix}$$

a vector in $\mathbb{R}^2$!

$$0\vec{v}_1 + \vec{v}_2 - \vec{v}_3$$

$\vdots$

Span

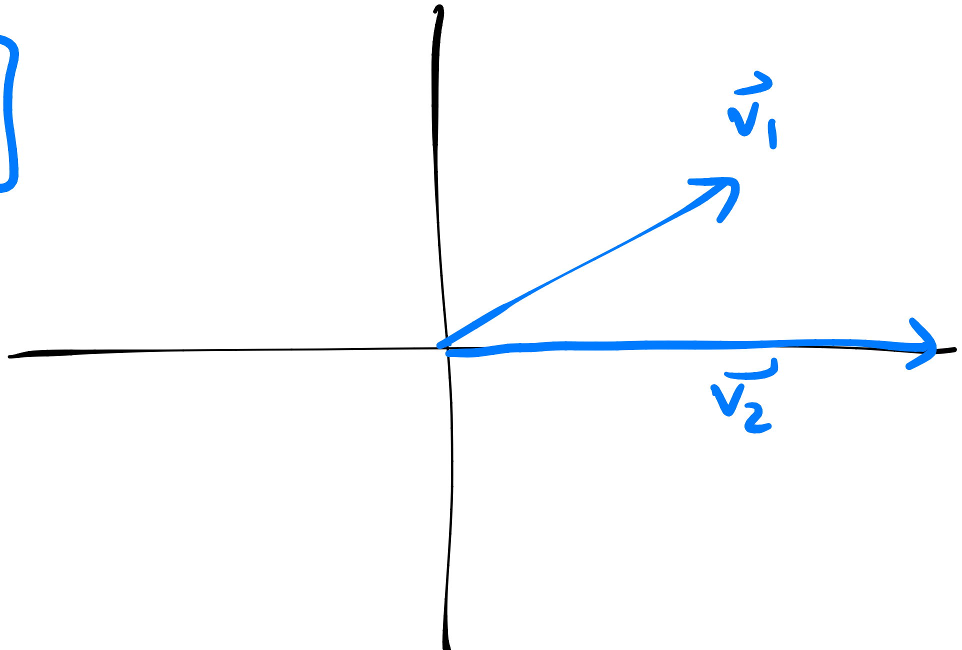
- Let $\vec{v}_1$, $\vec{v}_2$, ..., $\vec{v}_d$ all be vectors in $\mathbb{R}^n$.
- The **span** of $\vec{v}_1$, $\vec{v}_2$, ..., $\vec{v}_d$ is the set of all vectors that can be created using linear combinations of those vectors.
- Formal definition:

$$\text{span}(\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d) = \{a_1\vec{v}_1 + a_2\vec{v}_2 + \ldots + a_d\vec{v}_d : a_1, a_2, \ldots, a_d \in \mathbb{R}\}$$

Example

$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

$\vec{v}_1$ and $\vec{v}_2$ span all of $\mathbb{R}^2$!



We can! $\vec{v}_1$ and $\vec{v}_2$ aren't scalar multiples of each other: they point in diff. directions

Exercise

Let $\vec{v}_1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and let $\vec{v}_2 = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$. Is $\vec{y} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$ in $\text{span}(\vec{v}_1, \vec{v}_2)$?

If so, write $\vec{y}$ as a linear combination of $\vec{v}_1$ and $\vec{v}_2$.

$$w_1 \vec{v}_1 + w_2 \vec{v}_2 = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2w_1 \\ -3w_1 \end{bmatrix} + \begin{bmatrix} -w_2 \\ 4w_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{array}{l} 2w_1 - w_2 = 9 \\ -3w_1 + 4w_2 = 1 \end{array} \longrightarrow \text{solve for } w_1, w_2.$$