Dot products

in Overleaf:
\mathbb{R}

$\mathbb{R}$: real numbers

n: there are n real numbers in our vector.

Vectors

- A **vector** in $\mathbb{R}^n$ is an **ordered collection of n numbers**.
- We use lower-case letters with an arrow on top to represent vectors, and we usually write vectors as **columns**.

$$\vec{v} = \begin{bmatrix} 8 \\ 3 \\ -2 \\ 5 \end{bmatrix}$$

transpose

- Another way of writing the above vector is $\vec{v} = [8, 3, -2, 5]^T$.
- Since $\vec{v}$ has four **components**, we say $\vec{v} \in \mathbb{R}^4$.

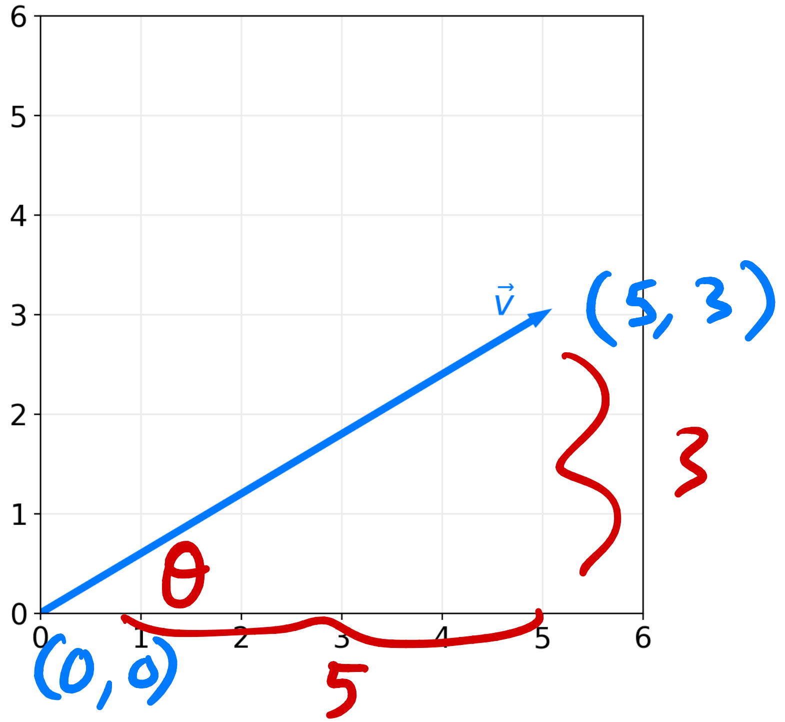
"elements" "in"

$$\vec{v} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\|\vec{v}\| = \sqrt{5^2 + 3^2} = \sqrt{34}$$

The geometric interpretation of a vector

- A vector $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ is an arrow to the point $(v_1, v_2, \dots, v_n)$ from the origin.

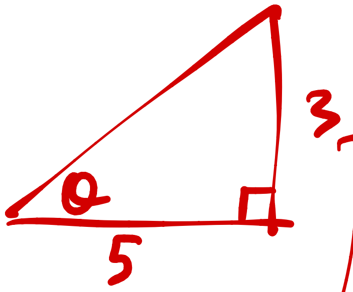


- The **length**, or L_2 **norm**, of $\vec{v}$ is:

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}$$

multidimensional Pythagorean theorem

- A vector is sometimes described as an object with a **magnitude/length** and **direction**.



$$\tan\theta = \frac{3}{5}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{3}{5}\right)}$$

both have the same number of elements

$\Rightarrow$ the same dimension

Dot product: coordinate definition

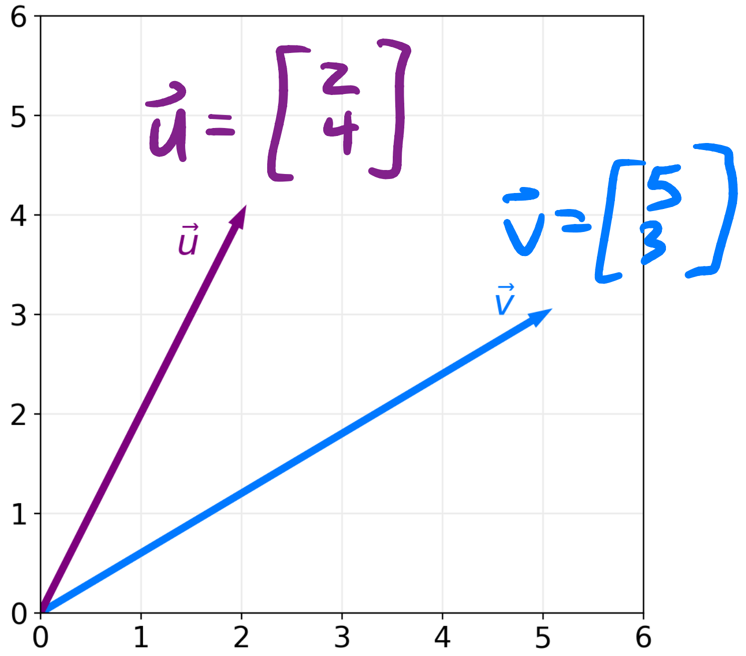
- The **dot product** of two vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^n$ is written as:

$$\vec{u} \cdot \vec{v} = \vec{u}^\intercal \vec{v}$$

- The computational definition of the dot product:

$$\vec{u} \cdot \vec{v} = \sum_{i=1}^{n} u_i v_i = u_1 v_1 + u_2 v_2 + \ldots + u_n v_n$$

- The result is a **scalar**, i.e. a single number.



$\vec{u} \cdot \vec{v} = (2)(5) + (4)(3) = 10 + 12 = \boxed{22}$ ← scalar! just one number!!!

$\vec{u}^T \vec{v} = \underset{1\times 2}{\begin{bmatrix} 2 & 4 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 5 \\ 3 \end{bmatrix}} = 22$

no dot!

match!

Question 🤔

Answer at q.dsc40a.com

Which of these is another expression for the length of $\vec{v}$?

- A. $\vec{v} \cdot \vec{v}$
- ~~B.~~ $\cancel{\sqrt{\vec{v}^2}}$
- C. $\sqrt{\vec{v} \cdot \vec{v}}$
- ~~D.~~ $\cancel{\vec{v}^2}$
- E. More than one of the above.

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$\vec{v}^2$ is undefined!

$\vec{v}_{n \times 1} \quad \vec{v}_{n \times 1}$

inner dimensions must match! but, $1 \neq n$.

$$\sqrt{\vec{v} \cdot \vec{v}} = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2} = \|\vec{v}\|$$